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Section 5.3: 12, 26c ("5 | a+b" meas that 5 divides a+b), 43, 44; ( 7th edition)

12.

Basis: f21 below = 1^2 = f1f2.

Inductive case: Suppose f21 + f22 + · · · + f2n = fnfn+1. Then

f21 + f22 + · · · + f2n + f2n+1 =fnfn+1 + f2n+1

=(fn + fn+1)fn+1

=fn+1fn+2

So f21 + f22 + · · · + f2n = fnfn+1, when n is a positive integer.

26.

c) Basis: Holds because 5 |(0 + 0 = 0)

Recursive: Suppose a + b = 5k for some integer k. Then 5|(a+2)+(b+3), because (a+2)+(b+3) = a+b+5 = 5k+5 = 5(k+1), where k+1 is also an integer. Similarly, 5|(a+3)+(b+2), because (a+3)+(b+2) = a+b+5 = 5k+5 = 5(k+1), where k + 1 is also an integer.

43. Basis: n(T) = 1 and h(T) = 0, and 1 >= 2\* 0 + 1

Recursive: Assume result holds for all binary trees smaller than T. Need to show that m(T) >= 2h(T) + 1 for the binary tree T. Based on the recursive definition of a full binary tree, T is formed by two subtrees T1 and T2, where T1 and T2 are smaller than T. By the induction hypothesis, we know that the inductive hypothesis holds for T1 and T2, i.e. n(T1) >= 2h(T1)+1 and n(T2) >= 2h(T2) + 1. By the recursive definition of n(T) and h(T), we have n(T) = 1 + n(T1) + n(T2) and h(T) = 1 + max(h(T1), h(T2)). We can then show that:

n(T) = 1 + n(T1) + n(T2)

>= 1 + 2h(T1) + 1 + 2h(T2) + 1

>= 1 + 2max(h(T1), h(T2)) + 2

= 1 + 2(max(h(T1), h(T2)) + 1)

= 1 + 2h(T)

44.

The base binary tree has a single node: its root. As it has no descendants, it is thought of as a leaf, giving # of leaves − # of internal nodes = 1.

More generally, suppose that this is true for a given pair of binary trees T1 and T2, and form a new tree T from these by attaching them to a new root node. This preserves all the old leaf nodes and adds one new internal node (the new root), giving:

(# of leaves of T) − (# of I.N.s of T)

= (# of leaves of T1 + # of leaves of T2) − (# of I.N.s of T1 + # of I.N.s of T2 + 1)

= (# of leaves of T1 − # of I.N.s of T1) + (# of leaves of T2 − # of I.N.s of T2) – 1

= 1 + 1 − 1 = 1,

using the inductive assumptions. By structural induction, this demonstrates the equality for all full binary trees.